121/1-MATHEMATICS PAPER 1
TIME: $2\frac{1}{2}$ HRS

NAME:……ADM NO……CLASS……

DATE……SIGN:…….

121/1
MATHEMATICS
PAPER 1
TIME: 2 ½ HOURS

INSTRUCTIONS TO CANDIDATES

1. Write your name, index number, class and school in the spaces provided above.
2. This paper consists of TWO sections I & II
3. Answer ALL the questions in section I and only FIVE questions from section II
4. All answers and working must be written on the question paper in the spaces provided below each question.
5. Show all the steps in your calculations giving your answers at each stage in the spaces below each question.
6. Marks may be given for correct working even if the answer is wrong.
7. Non-programmable silent electronic calculators and KNEC mathematical tables may be used except where stated otherwise.

FOR EXAMINERS USE ONLY

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL

SECTION I (50 MARKS)

Answer ALL Questions in this Section

1. Simplify $\frac{18ax-(3a-4x)(3a+4x)}{3a-8x}$ (3mks)

$(3a-4x)(3a+4x)$ is a difference of two squares and gives $9a^2-16x^2$

$18ax-(9a^2-16x^2)$

$18ax-9a^2+16x^2$

$16x^2+18ax-9a^2$

$16x^2+24ax-6ax-9a^2$

$8x(2x+3a)-3a(2x+3a)$

$\frac{(8x-3a)(2x+3a)}{3a-8x}=\frac{-1(3a-8x)(2x+3a)}{3a-8x}$

$=-2x-3a$

2. A rectangular floor of a room measures 5.4m long and 4.2m wide. The room is to be covered with square tiles. Calculate the minimum number of square tiles that can be used to cover the floor. (3mks)

Convert the metre into centimetres.

Find the GCD

2	540	420
2	270	210
3	135	105
5	45	35
	9	7

$2^2\times 3\times 5=60$cm

$=\frac{540}{60}\times\frac{540}{7}$

$=9\times 7$

$=63\ tiles$

3. Given the inequalities $x-5\leq 3x-8<2x-3$

(a) Solve the inequalities; (2mks)

$x-5\leq 3x-8$

$8-5\leq 3x-x$

$3\leq 2x$

$1.5\leq x$

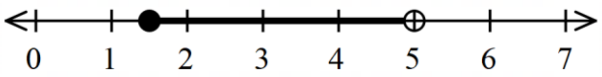
$3x-8<2x-3$

$3x-2x<8-3$

$x<5$

$1.5\leq x<5$

(b) Represent the solution on a number line. (2mks)



4. A watch looses 8 seconds every hour. It was set to read the correct time at 1100h on Sunday. Determine the time, in a 12-hour system, the watch will show on the following Thursday when the correct time is 0500h. (3mks)

Sun	1100h	
Mon	1100h	24hrs
Tue	1100h	24hrs
Wed	1100h	24hrs
Thur	0500h	18hrs
		90hrs

$90 \times 8 = 720\ sec$

$\frac{720sec}{60} = 12min$

$5.00am$

$-$ 12 minutes, and this gives

$4.48am$

5. An 890kg culvert is made of a hollow cylindrical material with outer radius of 76cm and an inner radius of 64cm. It crosses a road of width 3m. Determine the density of the material used in its construction in kg/m^3 correct to 1 decimal place. (3mks)

$\pi R^2h - \pi r^2h$

$\pi h(R^2 - r^2)$

$= \frac{22}{7} \times 3(0.76^2 - 0.64^2)$

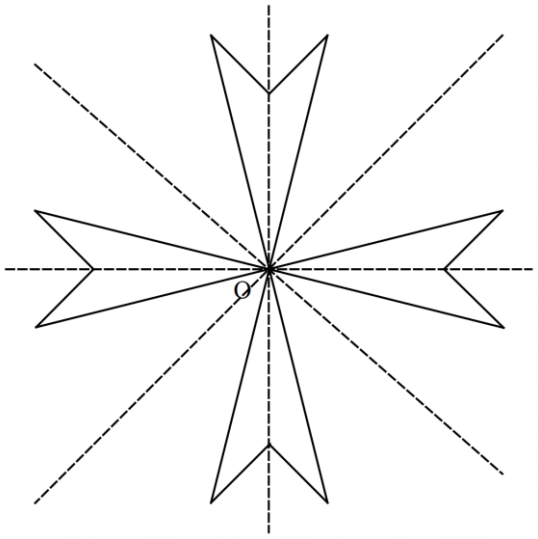
$= \frac{22}{7} \times 3 \times 0.168$

$= 1.584m^3$

$D = \frac{M}{V} = \frac{890}{1.584}$

$= 561.9kg/m^3$

6. The diagram below is part of a figure which has rotational symmetry of order 4 about the O.
   (a) Complete the figure. (2mks)
   (b) Draw all the lines of symmetry of the completed figure (2mks)



7. A line L is perpendicular to the line $\frac{2}{3}x + \frac{5}{7}y = 1$. Given that L passes through (4,11), find:

(a) Gradient of L; (1mk)

$21(\frac{2}{3}x + \frac{5}{7}y = 1)$

$14x + 15y = 21$

$y = -\frac{14}{15}x + \frac{7}{5}$

$m_1 = -\frac{14}{15}$

$m_2 = \frac{15}{14}$

(b) Equation of L in the form $y = mx + c$, where m and c are constants (2mks)

$\frac{y-11}{x-4} = \frac{15}{14}$

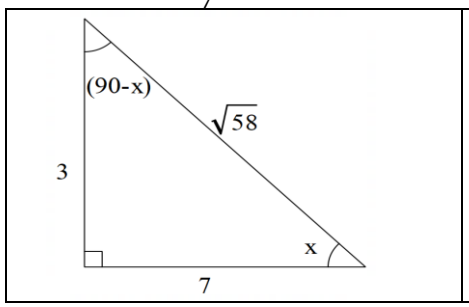
$14(y - 11) = 15(x - 4)$

$14y - 154 = 15x + 94$

$14y = 15x + 94$

$y = \frac{15}{14}x + \frac{47}{7}$

8. Given that $tanx^0 = \frac{3}{7}$, find $cos(90 - x)^0$, giving the answer in 4 significant figures. (3mks)



$c^2 = 3^2 + 7^2$

$c^2 = 58$

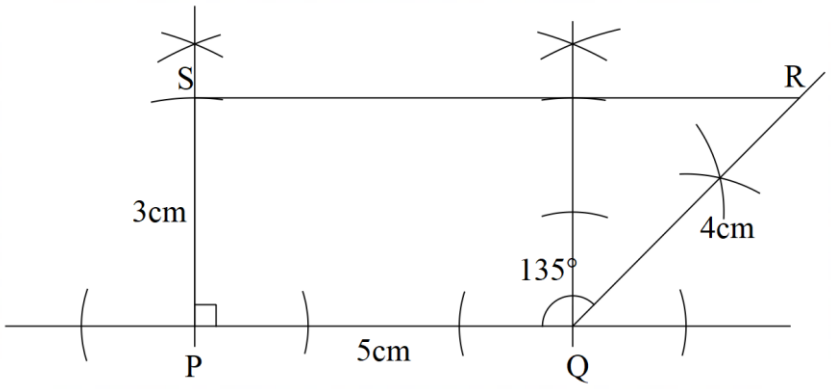
$c = \sqrt{58}$

$\cos(90 - x) = \frac{3}{\sqrt{58}}$

$= 0.3939$

9. Using a ruler and a pair of compasses only;

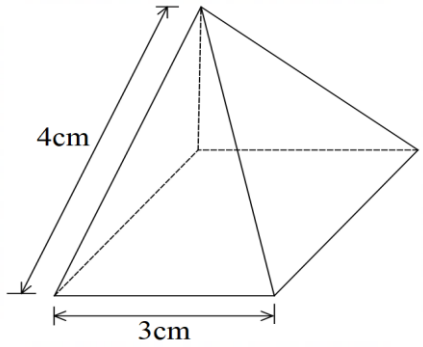
(a) construct a quadrilateral PQRS in which PQ=5cm, PS=3cm, QR=4cm, <PQR=135^0 and <SPQ is a right angle. (3mks)



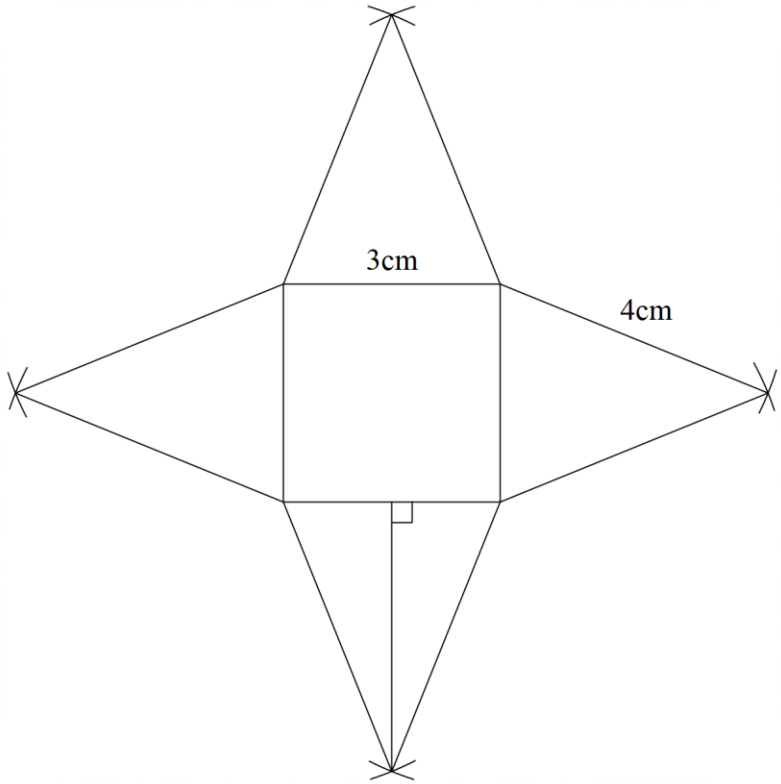
(b) The quadrilateral PQRS represents a plot of land drawn to a scale of 1:4000. Determine the actual length of RS in metres (1mk)

RS=$\frac{8\times4,000}{100} = 320m$

10. The figure below represents a right pyramid on a square base of side 3cm. the slant edge of the pyramid is 4cm.



(a) Draw a net of the pyramid (2mks)



(b) On the net drawn, measure the height of a triangular face from the top of the pyramid (1mk)
3.7cm

11. The size of two interior angles of an irregular polygon each measure 90^0. All the other remaining interior angles each measure 150^0. Determine the number of sides of the polygon. (3mks)

$90^0 + 90^0 + 150^0(n-2) = 180^0(n-2)$
$180^0 + 150^0 n - 300^0 = 180^0 n - 360^0$
$180^0 + 360^0 - 300^0 = 180^0 n - 150^0 n$
$180^0 = 30^0 n$
$n = 8 sides$

12. Solve for x in the equation $8^{x+1} - 2^{3x-1} = 120$ (3mks)

$2^{3(x+1)} - 2^{3x-1} = 120$

$2^{3x+3} - 2^{3x-1} = 120$

$2^{3x} \times 2^3 - \frac{2^{3x}}{2} = 120$

$Let\ P = 2^{3x}$

$2\left(8P - \frac{P}{2} = 120\right)$

$16P - P = 240$

$15P = 240$

$P = 16$

$2^{3x} = 2^4$

$3x = 4$

$x = 1\frac{1}{3}$

13. Convert $\frac{2\pi^c}{9}$ into degrees (2mks)

$\pi^c = 180^0$

$\frac{2\pi^c}{9} = \frac{2\pi^c}{9} \times \frac{180^0}{\pi^c}$

$= 40^0$

14. A businessman makes a profit of 20% when he sells a carpet for Kshs. 36,000. In a trade fair he sold one such carpet for Kshs. 33,600. Calculate the percentage profit made on the sale of the carpet during the trade fair. (3mks)

$120\% = 36{,}000$

$100\% = \frac{100 \times 36{,}000}{120}$

$= Kshs.30{,}000$

$Profit = 33{,}600 - 30{,}000$

$= Kshs.3{,}600$

$\%\ profit = \frac{3600}{30{,}000} \times 100$

$= 12\%$

15. By showing all the steps, use logarithms to evaluate: $\frac{5.627\times(0.234)^3}{(8.237)^{\frac{1}{2}}}$ (3mks)

Number	Std Form	Logarithm	
5.627	5.627×10^0	0.7503	0.7503
0.234	2.34×10^{-1}	$+\bar{1}.3692 \times 3$	$+\bar{2}.1076$
			$\bar{2}.8579$
8.237	8.237×10^0	$0.9158 \div 2$	-0.4579
0.02512	2.512×10^{-2}	$\bar{2}.4000$	$\bar{2}.4000$

16. Using the grid provided below, solve the simultaneous equations (3mks)

$x - 4y = -5$

$-x + 2y = 1$

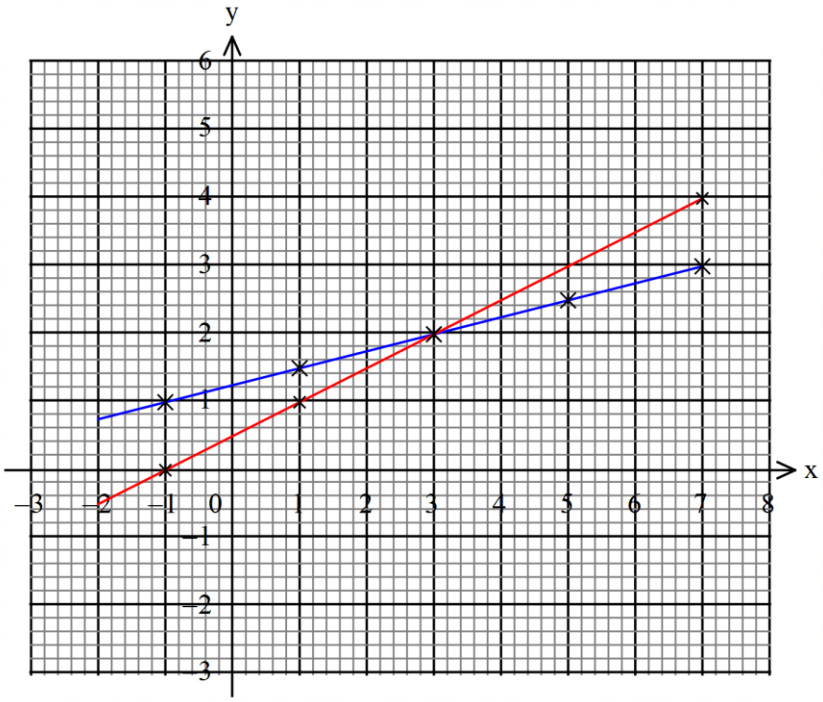
Solution

$x - 4y = -5$

x	-1	1	3	5	7
y	1	1.5	2	2.5	3

$-x + 2y = 1$

x	-1	1	3	5	7
y	0	1	2	3	4



$x = 3, y = 2$

SECTION II (50 MARKS)

Answer only five questions from this section

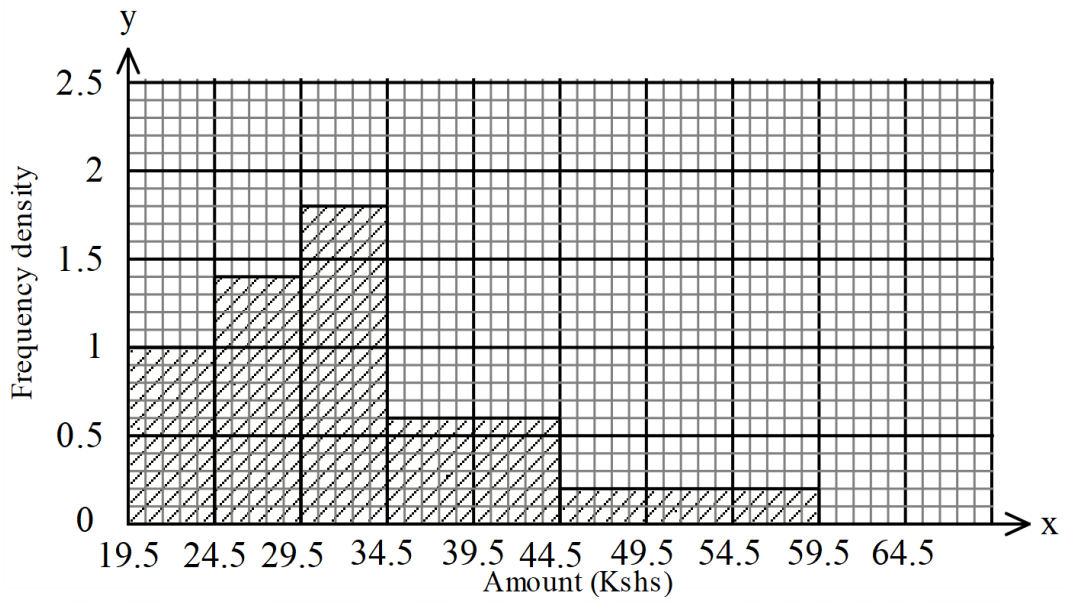
17. The amount of money, in Kenya shillings, spent on airtime by a group of 30 people in a period of an hour was recorded as shown below

27	20	21	24	22	25
42	34	55	26	30	39
35	46	32	21	38	34
31	37	27	29	32	56
33	44	25	31	28	30

(a) Complete the frequency distribution table below. (2mks)

Amount (Ksh)	20-24	25-29	30-34	35-44	45-49
Frequency	5	7	9	6	3
fd	1	1.4	1.8	0.6	0.2

(b) On the grid below, draw a histogram to represent the data. (4mks)



(c) Use the histogram to determine:

(i) The median amount of money spent on airtime by the 30 people. (2mks)

$1 \times 5 + 1.4 \times 5 + 1.8x = 15$

$x = 1.667$

$Median = 29.5 + 1.667 = 31.167$

(ii) The number of people who spent more than Kshs. 41.50 on airtime over that period (2mks)

$0.6 \times 3 + 0.2 \times 15 = 4.8$

$\approx 4people$

18. The average speed of a pick-up was $20km/h$ faster than the average speed of a lorry. The pick-up took 45 minutes less than the lorry to cover a distance of 180km.

(a) If the speed of the lorry was xkm/h:

(i) Write expressions in terms of x for the time taken by the lorry and the pick-up respectively to cover the distance of 180km (2mks)

$Lorry = \frac{180}{x}$

$Pick - up = \frac{180}{x+20}$

(ii) Determine the speed of the lorry and that of the pick-up. (4mks)

$\frac{180}{x} - \frac{180}{x+20} = \frac{45}{60}$

$\frac{180(x+20)-180x}{x(x+20)} = \frac{3}{4}$

$\frac{180x+3600-180x}{x^2+20x} = \frac{3}{4}$

$3600 \times 4 = 3(x^2 + 20x)$

$3x^2 + 60x - 14400 = 0$

$x^2 + 20x - 4800 = 0$

$x(x + 80) - 60(x + 80) = 0$

$(x - 60)(x + 80) = 0$

$Either\ x = -80\ (Ignore)$

$Or\ x = 60$

$\therefore Speed\ of\ the\ lorry = 60km/h$

$Speed\ of\ pick - up = 60 + 20$

$= 80km/h$

(b) The distance between towns A and B is 240km. On a certain day the pick-up started from town A at 8.30a.m. and the lorry started from town B at the same time. Determine the time that the lorry and the pick-up met. (4mks)

$Relative\ speed = 60 + 80 = 140km/h$

$Time = \frac{Distance}{Speed} = \frac{240}{140} = 1\frac{5}{7}hr$

$= 1hr\ 42\ \text{min}\ 51\ sec$

$8:30:00$

$+1:42:51$

$10:12:51\ am$

19. The position vector of point A and B are $\begin{pmatrix}-4\\6\end{pmatrix}$ and $\begin{pmatrix}-8\\2\end{pmatrix}$ respectively. Point M is the midpoint of AB and point N is the midpoint of OA.

(a) Find

(i) The vector AB (2mks)

$$\begin{pmatrix}-8\\2\end{pmatrix}-\begin{pmatrix}-4\\6\end{pmatrix}=\begin{pmatrix}-4\\-4\end{pmatrix}$$

(ii) The coordinates of points M and N (2mks)

$$\left(\frac{-4+-8}{2},\frac{6+2}{2}\right)=(-6,4)$$

$$\left(\frac{-4+0}{2},\frac{6+0}{2}\right)=(-2,3)$$

(iii) The magnitude of NM (2mks)

$$=\sqrt{(-6-(-2))^2+(4-3)^2}$$
$$=\sqrt{(-4)^2+(1)^2}$$
$$=\sqrt{17}=4.123$$

(b) The coordinates of a point C is $(2,a)$. vector CA is parallel to vector OB. Determine the value of a. (4mks)

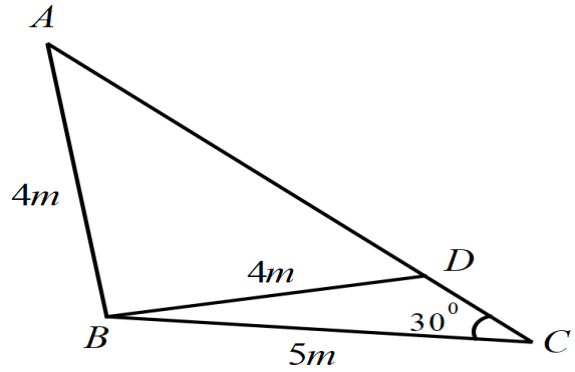
$$CA=OA-OC$$
$$=\begin{pmatrix}-4\\6\end{pmatrix}-\begin{pmatrix}2\\a\end{pmatrix}$$
$$=\begin{pmatrix}-6\\6-a\end{pmatrix}\quad k\begin{pmatrix}-6\\6-a\end{pmatrix}=\begin{pmatrix}-8\\2\end{pmatrix}$$

$$-6k=-8$$

$$k=\frac{4}{3}$$
$$6k-ak=2$$
$$6\left(\frac{4}{3}\right)-a\left(\frac{4}{3}\right)=2$$
$$\frac{4}{3}a=6$$

$$a=4.5$$

20. The figure below represents a triangular flower garden ABC in which AB=4m, BC=5m and <BCA=300. Point D lies on AC such that BD=4m and <BDC is obtuse.



Find, correct to 2 decimal places:

(a) <BDC; (2mks)

$\frac{4}{sin30^0} = \frac{5}{\sin D}$

$sinD = \frac{5sin30^0}{4}$

$\sin D = 0.625$

$D = 38.68^0$

$Obtuse = 141.32^0$

(b) The length of AD; (2mks)

$< ABD = 180 - (38.68 \times 2)$

$= 102.64^0$

$AD^2 = 4^2 + 4^2 - 2 \times 4 \times 4cos102.64$

$AD^2 = 39$

$AD = 6.24m$

(c) The length of DC; (2mks)

$< DBC = 180 - (30 + 141.32)$

$= 8.68^0$

$\frac{DC}{sin8.68} = \frac{4}{sin30}$

$DC = \frac{4sin8.68}{sin30}$ =1.21m

(d) The area of the flower garden ABC. (4mks)

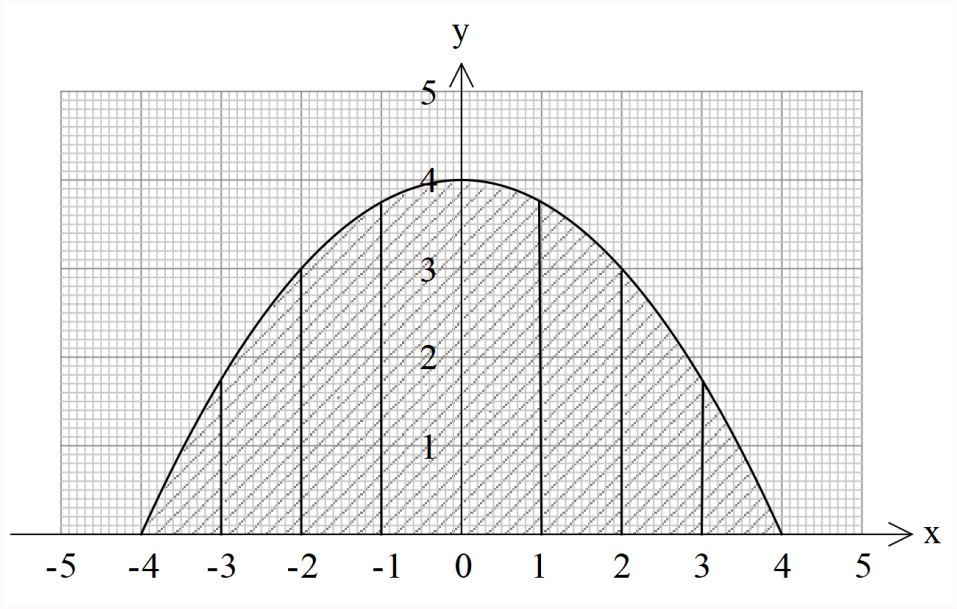
$AD = 6.24 + 1.21 = 7.45m$

$area = \frac{1}{2} \times 5 \times 7.45sin30^0$

$= 9.31m^2$

21. (a) On the grid provided, draw the graph of $y = 4 - \frac{1}{4}x^2$ for $-4 \leq x \leq 4$

x	-4	-3	-2	-1	0	1	2	3	4
y	0	1.75	3	3.75	4	3.75	3	1.75	0



(b) Using trapezium rule, with 8 strips, estimate the area bounded by the curve and the x-axis (4mks)

$A = \frac{1}{2}h\{(y_0 + y_8) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\}$

$A = \frac{1}{2}\{(0 + 0) + 2(1.75 + 3 + 3.75 + 4 + 3.75 + 3 + 1.75)\}$

$A = 21$

(c) Find the area estimated in part (b) above by integration (4mks)

$\int_{-4}^{4}\left(4 - \frac{1}{4}x^2\right)dx = \left[4x - \frac{x^3}{12} + c\right]_{-4}^{4}$

$= \left\{\left(4 \times 4 - \frac{4^3}{12} + c\right) - \left(4(-4) - \frac{(-4)^3}{12} + c\right)\right\}$

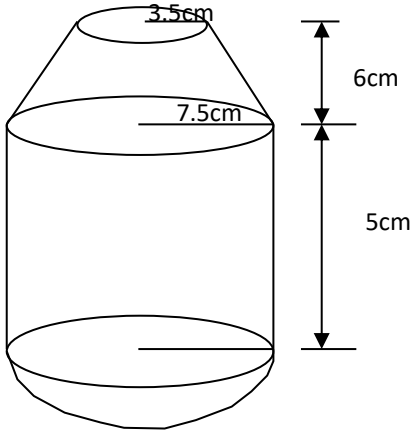
$= \left(10\frac{2}{3} - \left(-10\frac{2}{3}\right)\right) = 21\frac{1}{3}$

(d) Calculate the percentage error in estimating the area using trapezium rule (2mks)

$\frac{21\frac{1}{3} - 21}{21\frac{1}{3}} \times 100 = \frac{\frac{1}{3}}{21\frac{1}{3}} \times 100$

$= 1.5625\%$

22. A right conical frustrum of base radius 7.7cm, top radius 3.5cm and height 6cm is stuck onto a cylinder of base radius 7.7cm and height 5cm and further attached to the hemisphere to form a closed solid as shown below.



Find

(a) The full height of the solid (2mks)

$7.7 + 5 + 6 = 18.7cm$

(b) The volume of the solid (4mks)

$\frac{x}{3.5} = \frac{x+6}{7.7}$

$7.7x = 3.5x + 21$

$x = 5$

$\frac{1}{3}\pi(7.7^2 \times 11 - 3.5^2 \times 5)$

$\frac{1}{3}\pi(652.19 - 61.25)$

$\frac{1}{3} \times \frac{22}{7} \times 590.94$

$= 619.08cm^3$

$\pi R^2 H = \frac{22}{7} \times 7.7^2 \times 5$

$= 931.7cm^3$

$\frac{2}{3} \times \frac{22}{7} \times 7.7^3 = 956.55cm^3$

$Total = 2{,}507.33cm^3$

(c) The surface area of the solid. (4mks)

$\pi r^2 = \frac{22}{7} \times 3.5^2 = 38.5cm^2$

$\pi RL - \pi rl$

$\frac{22}{7} \times 7.7 \times 13.43 - \frac{22}{7} \times 3.5 \times 6.103$

$\frac{22}{7}(103.411 - 21.3605)$

$= 257.873cm^2$

$2\pi RH = 2 \times \frac{22}{7} \times 7.7 \times 5 = 242cm^2$

$2\pi r^2 = 2 \times \frac{22}{7} \times 7.7^2 \times 5 = 372.68cm^2$

$Total = 911.053cm^2$

23. Matrix M is given by $\begin{pmatrix} 4 & 7 \\ 5 & 5 \end{pmatrix}$

(a) Find M^{-1} (2mks)

$det = 4 \times 5 - 5 \times 7$

$= 20 - 35$

$= -15$

$M^{-1} = \frac{1}{-15}\begin{pmatrix} 5 & -7 \\ -5 & 4 \end{pmatrix} = \begin{pmatrix} -\frac{5}{15} & \frac{7}{15} \\ \frac{5}{15} & \frac{-4}{15} \end{pmatrix}$

(b) Sky-Scores High School purchased 8 bags of rice and 14 bags of sugar for Kshs 106,000. Excell High School purchased 10 bags of rice and 10 bags of sugar for Kshs 95,000. Each bag of rice cost Kshs R and a bag of sugar cost Kshs S.

(i) Form two matrix equations to represent the information above (2mks)

$8R + 14S = 106{,}000$

$10R + 10S = 95{,}000$

$4R + 7S = 53{,}000$

$5R + 5S = 47{,}500$

$\begin{pmatrix} 4 & 7 \\ 5 & 5 \end{pmatrix}\begin{pmatrix} R \\ S \end{pmatrix} = \begin{pmatrix} 53{,}000 \\ 47{,}500 \end{pmatrix}$

(ii) Use the matrix M^{-1} to find the prices of one bag of each item (6mks)

$\begin{pmatrix} -\frac{5}{15} & \frac{7}{15} \\ \frac{5}{15} & \frac{-4}{15} \end{pmatrix}\begin{pmatrix} 4 & 7 \\ 5 & 5 \end{pmatrix}\begin{pmatrix} R \\ S \end{pmatrix} = \begin{pmatrix} -\frac{5}{15} & \frac{7}{15} \\ \frac{5}{15} & \frac{-4}{15} \end{pmatrix}\begin{pmatrix} 53{,}000 \\ 47{,}500 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} R \\ S \end{pmatrix} = \begin{pmatrix} -\frac{5}{15} \times 53{,}000 + \frac{7}{15} \times 47{,}500 \\ \frac{5}{15} \times 53{,}000 - \frac{4}{15} \times 47{,}500 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -17{,}666\frac{2}{3} + 22{,}166\frac{2}{3} \\ 17{,}666\frac{2}{3} - 12{,}666\frac{2}{3} \end{pmatrix}$

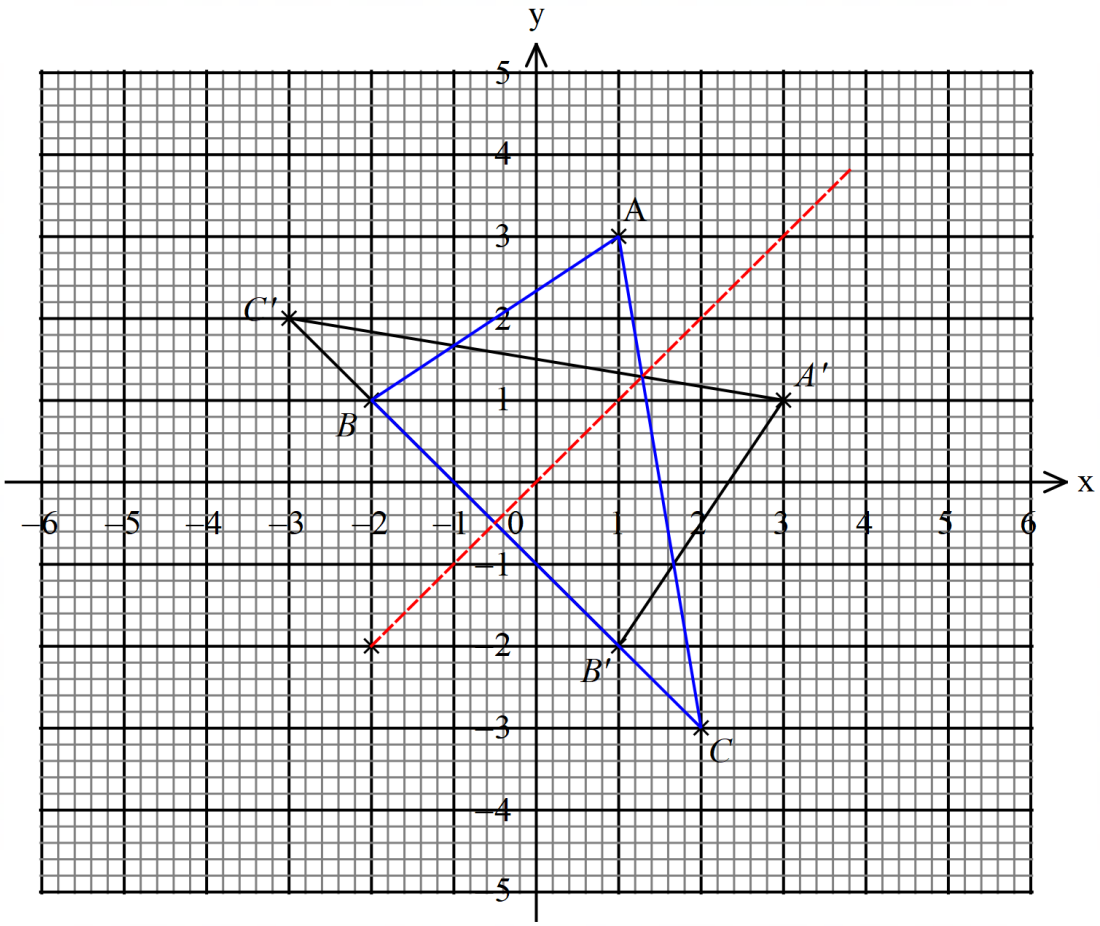
$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4{,}500 \\ 5{,}000 \end{pmatrix}$

Rice Kshs 4,500

Sugar Kshs 5,000

24. A (1,3), B (-2,1), C' (-3,2), B' (1, -2) and P (-2, -2) are points on the xy-plane.

(a) On the grid provided, plot A, B, B', C' and P (2mks)



(b) Triangle ABC is reflected onto triangle $A'B'C'$ through a mirror line M. Line M passes through P. Plot both triangle ABC and triangle $A'B'C'$ on the same grid (3mks)

(c) Find the equation of M in the form $ax + by + c = 0$ (3mks)

$gradient = \frac{-2-2}{-2-2} = \frac{-4}{-4} = 1$

$\frac{y-2}{y-2} = 1$

$y = x$

$x - y = 0$

(d) Write down the co-ordinates of C and A^{I} (2mks)

$C(2, -3)$

$A'(3,1)$